B.A./B.Sc. 4th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG4SEC21
(Vector Calculus)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any five questions: 5×2= 10

(a) Prove that $\frac{d}{dt}(\vec{r}.\frac{d\vec{r}}{dt}) = (\frac{d\vec{r}}{dt})^2 + \vec{r}.\frac{d^2\vec{r}}{dt^2}$ [2]

(b) Show that $\frac{d}{dt}(\vec{A}\times\vec{B}) = \vec{A}\times\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\times\vec{B}$ [2]

(c) Show that the following vectors are coplanar: [2]
$3\vec{a} - 7\vec{b} - 4\vec{c}, 3\vec{a} - 2\vec{b} + \vec{c}, \vec{a} + \vec{b} + 2\vec{c}$
where $\vec{a}, \vec{b}, \vec{c}$ are any three non-coplanar vectors.

(d) If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be unit vectors satisfying the condition $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = 0,$ then show that $\vec{\alpha}.\vec{\beta} + \vec{\beta}.\vec{\gamma} + \vec{\gamma}.\vec{\alpha} = -\frac{3}{2}.$ [2]

(e) Find a unit vector, in the plane of the vectors $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\left(\hat{i} + \hat{j} - 2\hat{k}\right)$, which is perpendicular to the vector $\left(2\hat{i} - \hat{j} + \hat{k}\right)$. [2]

(f) In any triangle ABC, with usual notations, prove that $c^2 = a^2 + b^2 - 2ab\cos C.$ [2]

(g) Show that the points (2,4,6), (3,4,5), (4,4,4) and (5,4,3) are coplanar. [2]

2. Answer any two questions: 2×5 = 10

(a) Let $f=x^3+y^3+z^3$; find the directional derivative of f at (1,-1,2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$ [5]

(b) Find grad f if $f=x^2+y^2$ and determine its magnitude and direction at (3,4). [5]

(c) Show that $\vec{\nabla}\times(\vec{\nabla}\times\vec{F}) = \vec{\nabla}(\vec{\nabla}.\vec{F}) - \vec{\nabla}^2\vec{F}$ [5]

(d) A particle moves along the curve $x= e^{-t}$, y = 2cost, z = 2sin3t. Determine the velocity and acceleration at any time t and their magnitude at t=0. [5]

3. Answer any two questions 2×10= 20

(a) (i) Prove, by definition of scaler product $\cos(\vec{A}+\vec{B}) = \cos\vec{A}\cos\vec{B} - \sin\vec{A}\sin\vec{B}$ [5]

(ii) Give the definition of vector product of two vectors. Find a unit vector perpendicular to each vector $\vec{\alpha} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\beta} = -2\hat{i} + \hat{j} - 2\hat{k}$ [5]

(b) (i) If $\vec{a}+\vec{b}+\vec{c}=0$,show that $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$ [5]

(ii) Given $\vec{f}.\frac{d\vec{f}}{dt} = 0$, prove that $\vec{f}$ is a constant. [5]

(c) (i) Show that the vector $\vec{V} = (4xy - z^3)\hat{i} + 2x^2\hat{j} - 3xz^2\hat{k}$ is irrotational. Also show that $\vec{V}$ can be expressed as the gradient of some scaler function ϕ. [5]

(ii) If $\vec{F}$ is a continuously differentiable vector point function such that $div\vec{F} = 0$, then there exists another vector point function $\vec{f}$ such that $\vec{F} = curl\ \vec{f}$. [5]

(d) (i) Find $div\ \vec{F}\ and\ curl\ \vec{F}\ where\ \vec{F} = (x^3 + y^3 + z^3 - 3xyz)$ [5]

(ii) Show that $\nabla^2\phi(r) = \phi''(r) + \frac{2}{r}\phi'(r)\ where\ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ [5]

B.A./B.Sc. 4th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG4SEC22
(Theory of Equations)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any five questions: 5×2 = 10

(a) Find the remainder when x^3 +3px+q is divided by x-α. [2]

(b) Form an equation of lowest degree with real coefficients having -2i as a root. [2]

(c) Form an equations of degree four with integral coefficients, where two of the roots are i and $\frac{1}{\sqrt{2}}$. [2]

(d) Show that the equation $3x^5$-$4x^2$+8=0 has at least two imaginary roots. [2]

(e) State Descartes' rule of sign for positive roots. [2]

(f) If α, β, γ be the roots of the cubic, $x^3 - px^2 + qx + r = 0$, find the value of [2]

$$\sum(\alpha - \beta)^2.$$

(g) Find the rational roots of $6x^4 - x^3 + x^2 - 5x + 2 = 0$ [2]

2. Answer any two questions $2\times5 = 10$

(a) Solve $x^3 - 6x - 9 = 0$, by Cardan's method. [5]

(b) Show that the roots of $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=\frac{1}{x}$, (where a>0, b>0, c>0) are all real. [5]

(c) If α, β, γ be the roots of the cubic, $ax^3 + 3bx^2 + 3cx + d = 0$, find the value of $(2\alpha-\beta-\gamma)(2\beta-\gamma-\alpha)(2\gamma-\alpha-\beta)$ [5]

(d) Reduce the reciprocal equation $x^5 - 6x^4 + 7x^3 + 7x^2 - 6x + 1 = 0$ to its standard form and solve it. [5]

3. Answer any two questions $2\times10 = 20$

(a) (i) If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, prove that each of them is equal to $\frac{6c-ab}{3a^2-8b}$. [5]

(ii) Reduce the equation $4x^4 - 85x^3 + 357x^2 - 340x + 64 = 0$ to a reciprocal equation and solve it. [5]

(b) (i) Solve the biquadratic equation $x^4 + 5x^3 + x^2 - 13x + 6 = 0$. [5]

(ii) Remove the fractional coefficients of the equation $2x^3 - \frac{3}{2}x^2 - \frac{1}{8}x + \frac{3}{16} = 0$. [5]

(c) (i) Show that the equation $x^3 - 2x - 5 = 0$ has no negative real root. [5]

(ii) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are β+γ, γ+α and α+β. [5]

(d) (i) Transform the equation $4x^4 + 3x^3 - 4x^2 - 5x + 2 = 0$ to one with unity as its leading coefficient. [5]

(ii) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4 + 16x^2 + 7x - 11 = 0$. [5]

B.A./B.Sc. 4th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG4SEC23
(Number Theory)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any five questions: $5\times2 = 10$

(a) Let *a*,*b* be two integers. If *a*|*b* and *b*|*a*, then prove that $a=\pm b$. [2]

(b) If n is even positive integer, then show that $\phi(2n)=2\phi(n)$. [2]

(c) Find the least positive residues in $2^{44}(\mathrm{mod}\,89)$. [2]

(d) If *gcd*(*a*,*b*)=1, then show that *gcd*(*a*+*b*,*a*-*b*)=1 or 2. [2]

(e) If $ax\equiv ay(\mathrm{mod}\,m)$ and *gcd*(*a*,*m*)=1, then prove that $x\equiv y(\mathrm{mod}\,m)$. [2]

(f) Find the remainder if $1!+2!+3!+\ldots+100!$ is divided by 15. [2]

(g) Find the number of integers less than 864 and prime to 864. [2]

2. Answer any two questions: $2\times5 = 10$

(a) Using division algorithm prove that the square of any integer is of the form 5k or $5k\pm1$, k is an integer. [5]

(b) If *gcd*(*a*,*m*)=1, then show that the linear congruence $ax\equiv b(\mathrm{mod}\,m)$ has unique solution. [5]

(c) Show that $a^{18}-b^{18}$ is divisible by 133 if a and both are prime to 133. [5]

(d) Define Euler's ϕ function. If $n=p_1^{a_1}p_2^{a_2}\ldots p_r^{a_r}$, where $p_1,p_2,\ldots,p_r$ are prime to each other then show that $\phi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_r}\right)$. [5]

3. Answer any two questions: $2\times10 = 20$

(a) (i) If $d=\gcd(a,b)$, then prove that $\frac{a}{d}$ and $\frac{b}{d}$ are integers prime to each other. [3]

(ii) If $\gcd(a,4)=2=\gcd(b,4)$, then show that If $\gcd(a+b,4)=4$. [2]

(iii) Find the general solution in integer of the equation If $7x+11y=1$. [5]

(b) (i) Prove that any positive integer is either 1 or prime or it can be expressed as the product of primes, the representation being unique except for the order of the prime factors. [6]

(ii) If p and p^2+8 be both prime numbers, then show that $p=3$. [4]

(c) (i) Find two integers u and v such that $\gcd(95,102) = 95u + 102v$. [5]

(ii) Solve the linear congruence $7x \equiv 3 (\text{mod}\, 15)$. [5]

(d) (i) Define totally multiplicative function. Give an example to show that if $f(n)$ is totally multiplicative then $\sum_{\frac{d}{n}} f(d)$ need not also be totally multiplicative. [2+3]

(ii) State and prove Möbius inversion theorem. [2+3]