B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG6DSE1B1
(Numerical Methods)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Explain the method of bisection for computing a real root of the equation $f(x) = 0$. What are its advantages and disadvantages? [3+2]

(b) (i) Deduce the condition of convergence of Newton Raphson iteration formula. [3]

(ii) The equation $x^2 + ax + b = 0$ has two real roots α *and* β. Show that the iteration method $x_{k+1} = -\frac{ax_k+b}{x_k}$ is convergent near $x = \alpha$ if $|\alpha| > |\beta|$. [2]

(c) Solve the following system of equations, [5]

$$x_1 + x_2 + 4x_3 = 9$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

by Gauss Seidel iteration method up to two decimal places.

(d) Derive Lagrange's interpolation formula when the function $f(x)$ is known to have (n+1) distinct values at the points $x_0, x_1, x_2, x_3, \dots, x_n$. [5]

(e) Prove that $\Delta^n(\frac{1}{x}) = \frac{(-1)^n.n!.h^n}{x.(x+h).(x+2h)....(x+nh)}$ for any positive integer n, Δ being forward difference operator and h being the step length. [5]

(f) (i) Find the relation between difference operator Δ and $D \equiv (\frac{d}{dx})$ of differential calculus. [2]

(ii) Estimate the missing term in the following table: [3]

x :	0	1	2	3	4
f(x) :	1	3	9	-	81

(g) Describe Trapezoidal rule for two points by using Newton's forward interpolation and the corresponding error. [2+3]

(h) Define order of convergence of an iterative method. Show that the following sequence have convergence of the second order with the limit $\sqrt{a}$, $x_{n+1} = \frac{1}{2}x_n(1 + \frac{a}{{x_n}^2})$ [1+4]

2. Answer any three questions: $10\times3 = 30$

(a) (i) Explain Euler's method for numerical solution of a first order differential equation $\frac{dy}{dx} = f(x,y)$ subject to the condition $y = y_0$ when $x = x_0$. [4]

(ii) Obtain Simpson's $\frac{1}{3}rd$ rule for numerical integration in composite form. Use this to calculate $\int_0^1 x^2(1-x)\,dx$ correct upto three decimal places, taking step length equal to 0.1. [3+3]

(b) (i) Establish Newton's Backward interpolation formula. [5]

(ii) Find the cubic polynomial which takes the following values, [5]

x :	1	3	5	7
y :	24	120	336	720

(c) (i) Describe the Regula-Falsi method for computing a real root of the equation $f(x) = 0$. Give the geometrical significance of this method. [5+2]

(ii) Obtain the Newton-Raphson iteration formula for computing $\sqrt{N}$, where N is a positive integer. [3]

(d) (i) Deduce the condition of convergence of fixed point iteration method. [5]

(ii) The equation $f(x) = 3x^3 + 4x^2 + 4x + 1 = 0$ has a root in (-1, 0). Determine the iterative function $\varphi(x)$ such that the iteration formula $x_{n+1} = \varphi(x_n)$, $x_0 = -0.5$, $n = 0,1,2, \ldots$ converges to the root. [5]

(e) (i) Solve the following system of equation by LU-decomposition method: [6]

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

(ii) Find by Euler's method, the value of y for $x = 0.3$ from the differential equation, $\frac{dy}{dx} = \frac{y-x}{y+x}$ taking step length 0.1, given that $y = 1$ when $x = 0$. [4]

B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMG6DSE1B2

(Complex Analysis)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) (i) Define the differentiability of a function of complex variable at a point. [2]

(ii) Prove that the differentiability of f at a point z_0 implies the continuity of the function at the same point. [3]

(b) Show that the function $u = cosx \cdot coshy$ is harmonic and find its harmonic conjugate. [2+3]

(c) State Laurent's theorem. Find the Laurent's series expansion of the function, $\frac{z^2-1}{(z+2)(z+3)}$, when $|z| \leq 2$. [2+3]

(d) Prove that a real valued function of a complex variable either has derivative zero or the derivative does not exist. [5]

(e) (i) Define bilinear transformation.

(ii) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively. [2+3]

(f) Prove that an analytic function with constant modulus in a region is constant. [5]

(g) Define radius of convergence of a power series. Find the radius of convergence of the power series $\sum \frac{z^n}{2^n+1}$. [2+3]

(h) Using contour integration, prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)^3} dx = \frac{\pi}{8a^3}$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) State and prove Liouville's theorem. [1+5]

(ii) Find the Taylor's series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1<|z|<2$. [4]

(b) (i) If *f(z)* is continuous in a region D and if the integral $\int f(z)dz$, taken round about any simple closed contour in D, is zero then prove that *f(z)* is an analytic function inside D. [5]

(ii) Show that both the transformations $w = \frac{z-i}{z+i}$ and $w = \frac{i-z}{i+z}$ transforms $|w| \leq 1$ into the upper half plane $I(z) \geq 0$. [5]

(c) (i) If *f(z)* is analytic within and on a simple closed contour C, and *a* is any point within C, then prove that $f'(a) = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^2}$. [5]

(ii) Let f be analytic in a region *G*. Prove the following: [2+3]

(i) If $f'(z)=0$ on G then *f* is constant.

(ii) If either Re *f* or Im *f* is constant on G, then *f* is constant on G.

(d) (i) Calculate the following integrals: [3+3]

(I) $\int_C \frac{e^z dz}{(z-1)(z+3)^2}$, where C is the circle $|z| = \frac{3}{2}$ and the integral is taken in the positive sense.

(II) $\int_C \frac{z^2+2z+1}{(z+1)^3} dz$, where C is the circle $|z| = 2$.

(ii) Show that the function $f(z) = z^3$ is analytic in a domain D of the complex plane *C*. [4]

(e) (i) Consider the function f defined by, [3+3]

$f(z) = \left(\frac{x^3-y^3}{x^2+y^2}\right) + i\left(\frac{x^3+y^3}{x^2+y^2}\right)$, when $z \neq 0$

= 0, when z=0.

Show that the function *f* satisfies the Cauchy-Riemann equations at the origin, but *f* is not differentiable at *z=0.*

(ii) If a function of complex variable *f* is differentiable at a given point, then give an example to support that $|f|$ may not be differentiable at the same point. [4]

B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG6DSE1B3
(Linear Programming)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) (i) Define a convex set. [2]

(ii) Show that the hyper plane is a convex set. [3]

(b) Show that the dual of the dual is the primal itself. [5]

(c) Solve the following problem graphically, [5]

$$\text{Max } Z = 6x_1+10x_2$$
$$\text{subject to } 3x_1+5x_2 \le 10$$
$$5x_1+3x_2 \le 15$$
$$\text{and } x_1, x_2 \ge 0$$

(d) Food x contains 6 units of vitamin A and 7 units of vitamin b per gram and costs 12p/gm. Food y contains 8 units and 12 units of A and B per gram respectively and costs 20p/gm. The daily requirements and vitamin A and vitamin B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P to minimize the cost. [5]

(e) (i) State the fundamental theorem of duality. [3]

(ii) Show that the set, $X = \{(x_1,x_2): x_1^2+x_2^2= 16\}$ is not a convex set. [2]

(f) Use two phase simplex method to solve the problem [5]

$$\text{Maximize } Z= 2x_1+x_2+x_3$$
$$\text{Subject to } 4x_1+6x_2+3x_3 \le 8$$
$$3x_1-6x_2-4x_3 \le 1$$
$$2x_1+3x_2- 5x_3 \ge 4$$
$$\text{and } x_1, x_2, x_3 \ge 0$$

(g) Solve the following system of linear simultaneous equations using the Simplex method, $x_1+x_2=1$, $2x_1+x_2=4$ [5]

(h) Solve the following problem by simplex method [5]

$$\text{Max } Z = 3x_1+x_2+3x_3$$
$$\text{Subject to } 2x_1+x_2+3x_3 \le 2$$
$$x_1+2x_2+3x_3 \le 5$$
$$2x_1+2x_2+x_3 \le 6$$
$$x_1, x_2, x_3 \ge 0$$

2. Answer any three questions: $10 \times 3 = 30$

(a) (i) Solving the dual problem, obtain the optimal solution of [7]

$$\text{Min } Z = 4x_1 + 3x_2$$
$$\text{Subject to } x_1 + 2x_2 \geq 8$$
$$3x_1 + 2x_2 \geq 12,$$
$$x_1, x_2 \geq 0$$

(ii) Determine the position of the point (1,-2, 3,4) relative to the hyperplane, [3]

$4x_1 + 6x_2 + 2x_3 + x_4 = 2$

(b) (i) Show that the set of all feasible solutions of the system $Ax=b$, $x \geq 0$ is a convex set. [5]

(ii) Solve graphically, [5]

$$\text{Min } Z = 2x_1 + 3x_2,$$
$$\text{Subject to } 2x_1 + 7x_2 \geq 22$$
$$x_1 + x_2 \geq 6$$
$$5x_1 + x_2 \geq 10$$
$$\text{and } x_1, x_2 \geq 0$$

(c) (i) Verify graphically whether the following problem has bounded or unbounded solution, [6]

$$\text{Max } Z = 3x_1 + 2x_2$$
$$\text{Subject to, } x_1 \leq 3,$$
$$x_1 - x_2 \leq 0,$$
$$\text{and } x_1, x_2 \geq 0.$$

(ii) Define hyperplane. Show that $X = \{x: |x| \leq 2\}$ is a convex set. [2+2]

(d) Using Big-M method, solve the following the problem, [10]

$$\text{Maximize } Z = 8x_2,$$
$$\text{Subject to } x_1 - x_2 \geq 0$$
$$2x_1 - 3x_2 \leq 6,$$
$$x_1, x_2 \text{ are unrestricted in sign.}$$

(e) Let $x_1=2$, $x_2=3$, $x_3=1$ be a feasible solution of the following LPP. [10]

$$\text{Max } Z = x_1 + 2x_2 + 4x_3$$
$$\text{Subject to } 2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$
$$\text{and } x_1, x_2, x_3 \geq 0.$$

Reduce the above feasible solution to basic feasible solutions.